Playing "Pythagoras" in Padua and Florence: a Sixteenth-Century *Rithmomachia* manuscript at the University of Pennsylvania

Jugar a "Pitàgores" a Pàdua i Florència: un manuscrit de la *Rithmomachia* del segle XVI a la Universitat de Pennsilvània

Jugando "Pitágoras" en Padua y Florencia: un manuscrito de la *Rithmomachia* del siglo XVI en la Universidad de Pensilvania

Brincando de "Pitágoras" em Pádua e Florença: um manuscrito de *Rithmomachia* do século XVI na Universidade da Pensilvânia

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Abstract: A manuscript in the Lawrence J. Schoenberg Collection at the University of Pennsylvania Libraries (UPenn LJS 232) contains a manual for the medieval game rithmomachia by Carlo di Ruberto Strozzi, preceded by a brief treatise on proportion by Benedetto Varchi, both in vernacular; they were inspired by the Latin publication of Jacques Lefèvre d'Etaples. An examination of the treatise and the circle of learned Florentines involved in its production offer an example both of the ways that the game spread in European university cultures, and the limits of interest in the Boethian mathematics of proportion that the game was intended to exercise.

Keywords: *Rithmomachia – Pythagoreanism – History of Education – History of Mathematics –* Universities – Florence – Padua.

Resumen: Un manuscrito de la Colección Lawrence J. Schoenberg de las Bibliotecas de la Universidad de Pensilvania (UPenn LJS 232) contiene un manual del juego medieval *Rithmomachia* de Carlo di Ruberto Strozzi, precedido de un breve tratado sobre la proporción de Benedetto Varchi, ambos en lengua vernácula; se inspiraron en la publicación en latín de Jacques Lefèvre d'Etaples. El examen del tratado y del círculo de sabios florentinos implicados en su producción ofrece un ejemplo tanto de las formas en que el juego se difundió en las culturas universitarias europeas, como de los límites del interés por las matemáticas boetianas de la proporción que el juego pretendía ejercitar.

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Rithmomachia was a game with a venerable pedigree, though not quite so venerable as some of its partisans claimed, and with an elevated social status. It arose in the eleventh century as an educational aid for the developing curriculum of the liberal arts. In particular, it employs the mathematics of proportion as described in the *Arithmetic* and *Music* of Boethius (ca. 480-524), two foundational texts of the mathematical section of the liberal arts—the quadrivium—that took shape during that era. These two Boethian works were themselves Latin versions of writings by the Neo-Pythagorean philosopher Nicomachus of Gerasa (ca. 60-ca. 120). Early manuscripts attest to the game's spread among monastic schools and later, in those university communities where the curriculum included Boethius. Indeed, the game was never listed among regular leisure-time games or activities such as dice-playing or even chess, but only in these educational contexts; it was too complicated for those without that particular sort of mathematical knowledge to play or to enjoy.

Thus, while the game is interesting on its own merits, its history also contributes in its small way to the history of mathematics education. Since it followed the use of Boethius in the classroom, its presence serves as a sort of marker for the study of those works, a subject about which still too little is known. The particulars of the game's rules might vary somewhat over time and place, but the basic features remained the same. It called for two players and a rectangular checkered board generally the size of two chess boards, a game on which it was clearly based. Each side has 24 playing pieces. These pieces are distinguished by shape (standardized early) and by the numbers written on them: 8 circles, 8 triangles, 7 squares, and a single stepped pyramid. (fig. 1) The sides are distinguished as

Odd and Even; the numbers on the pieces are generated by various operations on the first four odd and even numbers respectively: 3-5-7-9 (for Boethius, 1 is the unit and not a number) and 2-4-6-8. Players move one piece per turn, as with chess. Rules define the ways in which a given piece could move (1, 2, or 3 squares each for circles, triangles and squares), the strategies for the capture of pieces, and the final victory.[2]

The game enjoyed a new wave of popularity in the sixteenth century; a number of printed editions appeared across Europe, and manuscript versions circulated as well. Yet by the end of the century, both the game and Boethius's treatises disappeared permanently, victims of long-term changes in mathematics education as well as the field of mathematics more generally. One of the manuscripts from that final era is at the University of Pennsylvania, part of the Lawrence J. Schoenberg and Barbara Brizdle Manuscript Initiative. It offers a fascinating view of the environment in which the game flourished during those last decades. Boethius's Neopythagorean philosophy and the importance he placed on the study of number in the formation of character and morals still held interest for some people, while others found useful his discussions of mathematical proportion. The last appearance of game manuals indicates the locations of that last wave of interest.

UPenn LJS 232 consists of 60 paper folios written in brown ink with red rubrics and a few blue initials; it includes six diagrams and drawings.[3] It seems to have been copied, in a humanistic hand and with youthful enthusiasm (as seen, for example, in many very extended letters such as s) from one of the Florentine copies. The hand is that sof the otherwise unattested Marchiono de Marchioni, who identifies himself as copyist (Upenn LJS 232, fol. 3r)[4] and in Latin as owner ("Hic liber est mei Marci Antonii, fol. 1r). The

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[2] For a fuller discussion of the game and its history, see Ann E. Moyer, *The Philosophers' Game: Rithmomachia In Medieval and Renaissance Europe: with an Edition of Ralph Lever and William Fulke, The Most Noble, Auncient, and Learned Playe (1563)* (Ann Arbor: University of Michigan Press, 2001).

[3] *Transformation of knowledge: Early Manuscripts from the Collection of Lawrence J. Schoenberg*, ed. Crofton Black (London: Paul Holberton, 2006), 45-46.

[4] Parenthetical citations refer to this manuscript unless otherwise stated.

manuscript had been part of the library of Giannalisa Feltrinelli. It was auctioned by Christie's with the rest of her collection in 1997, when it was acquired by Lawrence J. Schoenberg. Schoenberg and Barbara Brizdle deposited it at the University of Pennsylvania in 2011; Penn acquired it in 2019 as a gift of Barbara Brizdle Schoenberg. Several folios, especially at the beginning, suffer from bleed through and ink damage via oxidation. Six other manuscript copies of this set of works are known (in whole or in part), five of them in Florence.[5]

The rithmomachia manual presented in this manuscript is a composite, two different works in vernacular written in 1539-40 by two authors who knew each other well, each with its own introduction. First is a treatise on proportions and proportionality by Benedetto Varchi. That is followed by a dialogue on the game by Carlo di Ruberto Strozzi; appended at the end is an anonymous little set of game rules in Latin along with related brief passages. The component sections of the work, the authors, and the communities of which they were a part illustrate the network of connections that allowed the game to spread, the range of approaches to mathematics in education, and the limits of both the game and of Boethius's arithmetic in their last decades.

Benedetto Varchi (1503-65) was a prominent Florentine man of letters, known as a historian, philosopher, and poet. At the time he wrote the treatise on proportion he was living in Padua as an exile, having briefly supported the rebellion that took up arms after the assassination of Alessandro de' Medici and the succession of Cosimo. He had been actively engaged with the intellectual culture of the university cities of Padua and Bologna while working both formally and informally as tutor. This work had associated him further with the rebel exiles; it had begun in Venice and then Padua with Ruberto Strozzi, son of

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[5] Florence, Biblioteca Nazionale Centrale (BNCF) Landau Finaly 205; BNCF Mag. VIII. 1492 (Treatise on proportion only); BNCF Mag. XI. 125; BNCF Mag. XI. 135; Florence, Biblioteca Riccardiana. Ms. 890. Apparently autograph; imprimatur 130r. Modena, Biblioteca Estense. Campori 501.

the exile leader Filippo. Filippo was captured after leading the exiles to defeat at the battle of Montemurlo. Varchi continued to work with the younger Strozzi sons until May 1538.[6]

Varchi would remain in the university towns of Padua and Bologna (and briefly, Ferrara) until 1543, returning to Florence at the invitation of Duke Cosimo. After he left the service of the Strozzi he shared a residence in Padua with the Florentine Albertaccio del Bene.[7] He resumed his correspondence with Luca Martini in Florence and began working on translations from Greek and Latin into vernacular.[8] In 1539 Varchi began offering academic advice and support via letters to Carlo di Ruberto Strozzi, a distant relative of Filippo's branch of the family;[9] Varchi's surviving letters begin in late August but refer to a correspondence begun earlier.[10] He offered advice at some length on authors Carlo should read, and hoped to persuade him to move from Florence to Padua to continue his studies.

Varchi's Florentine correspondents also included Cosimo Rucellai. He wrote to Rucellai in 1539 with a request that he send Varchi a game set and manual for rithmomachia, which he recalled had been in his family's possession. Cosimo responded in late June with a query about how best to send them on.[11] Eventually they arrived, and soon Varchi, Albertaccio, and their friend Ugolino Martelli were avid players, as Varchi wrote later in a letter that served to introduce his manual.

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[6] Richard Samuels, "Benedetto Varchi and Sixteenth-Century Florentine Humanism," Ph.D. thesis, University of Chicago, 1976, 168, 180. 193.

[7] Samuels, "Benedetto Varchi," 196.

[8] Samuels, "Benedetto Varchi," 199-202.

[9] Samuels, "Benedetto Varchi," 206.

[10] Letters 37-42 and 44-47, in Benedetto Varchi, *Lettere, 1535-1565,* ed. Vanni Bramanti, Studi e testi del Rinascimento europeo (Rome: Edizioni di storia e letteratura), 2008.

[11] Cosimo Rucellai to Benedetto Varchi June 28, 1539 (letter 38), 140-42, in Lettere *a Benedetto Varchi, 1530-1563*, ed. Vanni Bramante, *Cinquecento Testi*, vol. 18 (n. s., 4) (Manziana (Rome): Vecchiarelli, 2012).

In many ways, this little group resembled the game's traditional environment. Sixteenth-century Padua was perhaps the quintessential university town; universities had long ago replaced monasteries as the principal environment for the game. Varchi was in residence with friends and actively involved in the intellectual community. Yet apparently rithmomachia was not a part of that community; when Varchi wanted to take up the game, he did not find players, manuals, or game sets in Padua. Rather, he wrote to a fellow Florentine back home, where he himself had learned to play it.

Florence itself had not really been a university town for some time, though of course it was famous as a center of learning and especially of humanistic study. Since 1473 its university had been merged with that of Pisa and primarily located there, though humanities professors had often remained in Florence. Varchi's familiarity with the game had not come from that university setting at all; rather, its source was a learned circle, a sort of early academy, associated with Cosimo Rucellai's family. Cosimo's grandfather, Bernardo Rucellai, had established the garden at the family home in Florence as a regular gathering spot for learned conversation that attracted both Florentines and visitors; that tradition had been maintained by his sons well into the 1520s and as the present manuscript attests, into the 1530s. It had provided a forum for men such as Niccolò Machiavelli to present and discuss their work, and many others recalled later in life how important those gatherings at the Rucellai gardens had been to their own intellectual formation.

Most of the information about Florentines and rithmomachia comes from the two introductory letters in this composite work. Carlo di Ruberto Strozzi recounts the game's Florentine history. The letter is addressed to Niccolò Alamanni, one of the sons of Luigi Alamanni. Strozzi states that he is sending this work together with a set of playing pieces from Luca Martini, as well as Varchi's treatise on proportions and proportionality. He suggests that since Niccolò and his brother Battista had been to Paris they might already be familiar with it (fol. 26v).

Then Strozzi recalls the occasion that serves as the setting of the dialogue. He had gone alone to the Rucellai gardens. Martini was not there; it had been the last Thursday of Carnival, the Berlingaccio de la luna, and Martini had gone to Fiesole with another friend, Filippo Guadagni (fol. 26v).[12] At the garden he encountered Cosimo Rucellai speaking with Jacopo Vettori, showed them Varchi's treatise, and asked Cosimo to explain about the game. These three figures would serve as the interlocutors of the dialogue.

Rucellai then recounted how the game had arrived in the city some years earlier. A game set and a set of rules copied in Latin had been sent to his father, Palla di Bernardo Rucellai, from someone studying in Paris (fol. 27v). It had come, he said, from the school of Jacques Lefèvre d'Etaples (c. 1455-c. 1536).[13] Palla had hoped that his sons would learn to play it. The game was modern but dealt with ancient topics; it was intended to help youth restore themselves when they were exhausted from study yet also pass their leisure time productively. Yet no one in the city knew anything about it or the mathematical principles behind it, and it was simply buried, according to Carlo Rucellai, for several years.

A few years later, Palla had described the game to Benedetto Varchi. Varchi had borrowed the materials and learned to play, and then returned with his friends Lorenzo Lenzi and Marcello Cervini (later to become a cardinal and then, briefly, Pope Marcellus) to play with Palla; eventually he did indeed teach it to Palla's sons.[14] When Varchi wrote to Cosimo Rucellai in 1539 from Padua, Cosimo certainly remembered the game, along with the name Varchi had used for it, "Pythagoras," though Rucellai noted in his letter that they had not kept it up.[15]

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[12] Thus, Strozzi sets his dialogue at the very end of January; Ash Wednesday fell on February 5, 1540.

[13] Jordanus Nemorarius, Jacques Lefèvre d'Étaples, and Boethius, *In hoc opere contenta. Arithmetica decem libris demonstrata. Musica libris demonstrata quattuor. Epitome in libros arithmeticos divi Severini Boetii. Rithmimachie ludus qui et pugna numerorum appellat (*Paris: Joannes Higmanus et Volgangus Hopilius, 1496); (rpt. Paris: Henricus Stephanus, 1514).

[14] Richard Samuels suggests it was sometime after 1526. Samuels, "Benedetto Varchi," 218.

[15] *Lettere a Benedetto Varchi*, Letter 58, 140.

This case offers an example of how rithmomachia might travel informally from one place to another. It was not unusual for Florentines to study in Paris, and it is not surprising that books and more might accompany on occasion the letters that students exchanged with family and friends. Lefèvre's many educational publications had included mathematics textbook; this volume was a collection of several works that contained a game manual and represented part of the curriculum he helped develop and implement at the College du Cardinal Lemoine. Lefèvre himself had died a few years earlier, widely respected as a humanist educator and religious thinker. Rucellai noted that the man was "as you may know, of greatest fame in the field of the liberal arts, and of the holiest of habits".[16]

Florentines could also point to a local connection with Lefèvre; he had visited Florence some years earlier, part of his travels in Italy in 1491-92. The time Lefèvre spent with Marsilio Ficino and Giovanni Pico della Mirandola had left a singular impression on him as well as upon the Florentines. He had returned to Paris, where he remained at the College du Cardinal Lemoine from 1494 until 1507, developing the curriculum, with its emphasis on mathematics and moral development, of which the game had been a part.

Although both the College and the Rucellai gardens served as centers of learned conversation and exchange, they differed in significant ways. The Florentines who frequented the Orti Oricellari, as they referred to it, discussed a range of learned humanistic topics including history, politics, and letters. Though the group might well include youths who were still involved in studies, the writings, issues, and ideas discussed in its gatherings were not, nor were they intended to be, about topics or texts connected with university work. Nor could such groups of students be found elsewhere in the city; students of natural philosophy, medicine, and mathematics were at Pisa. Thus, it was not surprising that the game did not strike deep roots upon its arrival in Florence. It came and went during those

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[16] "come potete havere inteso di grandissima fama ni gl' luogo nelle discipline liberali, et di santità di costumi." Fol. 26v.

years as a pastime for a few and no more, though the Rucellai kept both the game set and the manual.

Varchi's request of 1539 did nonetheless lead to a new flurry of interest in the game in Florence, when Cosimo Rucellai discussed the query with his friends. Varchi's introductory letter to his text on proportion is composed as a letter to Luca Martini dated the last day of December 1539. This was the mailing that had sent Strozzi off to the Rucellai gardens, alone because Martini had gone elsewhere for a carnival celebration. Varchi alludes to the past events as described by Strozzi and his interlocutor Rucellai. He reported that once they had received the game set and rules in Padua, they were playing the game often, almost daily, "We play often here at home, and nearly every day, to the extent that Albertaccio and Ugolino—you know how they ordinarily loathe and despise almost all other games—have become real professionals."[17]

Martini, notes Varchi, is not the only Florentine who has expressed interest; the artists Niccolò Tribolo and Agnolo Bronzino as well as their acquaintance Lasagnino hoped to learn as well.[18] Yet they were stymied due to their ignorance of the study of proportion. No one in their circle was familiar with the subject, and apparently no one in Florence was currently playing the game. They all hoped that Varchi could explain the mathematical principles for them. These artists were presumably familiar with the principles of their Florentine predecessor Leon Batista Alberti, who advocated study and application of mathematical proportion in painting and architecture; they would seem to have been a

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[17] "Noi giuochiamo qui in casa spesse volte, et quasi ogni giorno di modo che messer Albertaccio et messer Ugolino, che sapete quanto hanno in odio ordinariamente et in abbominatione gli altri giuochi quasi tutti, fanno di questo professione apertissimamente ..." fol. 1v. See also Samuels, "Benedetto Varchi," 219; Samuels uses Florence, BNF Naz. 2,2: 278.

[18] Lasagnino's identity is unclear. Silvestro Ganassi mentions a Lodevico Lasagnino Fiorentino who played the viola da gamba in his *Regula Rubertina* (Venice 1542-3), 2.16; Luigi Lasagnino of Florence is noted as a violinist in Giuseppe Branzoli, *Manuale storico del violinista* (Florence: Venturini, 1894), 59.

natural audience for the study of proportion. Here they did indeed express interest, but admitted no familiarity.

Varchi noted that in fact there were several people in Florence who could offer instruction in the mathematics of proportion as well as he could or better. He mentions a Fra Giuliano del Carmine, "your" Don Mauro, Antonio degli Alberti, and Papi Tedaldi, and says there are many others as well.(2r) Antonio degli Alberti (1495-1555), who did indeed briefly teach mathematics along with ethics and philosophy while studying in Pisa, had frequented the Rucellai gardens from the early years.[19] Tedaldi was also a Florentine man of letters; Anton Francesco Doni would cast him as one of the interlocutors in his dialogue *I marmi* (1552). Despite these evident connections, the subject itself seems not to have been current.

Varchi continued that while he was happy to comply with their request to write on proportion and proportionality sufficient to understand the principles at work in the game, he was too busy to write about the game as well. He suggested that Martini or Strozzi check in with Cosimo Rucellai, since Rucellai had learned the game from Varchi years ago; that is the point at which Strozzi begins his own story in his introduction to the game description. The two were clearly working together on their composite manual. Both used Lefevre's volume as their point of departure. Yet their work also showed distinct differences from that of Lefèvre, differences that highlight some divergences between their educational goals and his. In particular, Lefèvre connected mathematics education with character development and moral education, a feature that is absent from the work of Varchi and Strozzi.

Varchi sets the tone as the author of the volume's first half. He has based his little treatise on a number of authors, he says, but particularly Boethius and Euclid (fol. 2r). Since the game had been based on Boethius's treatise, that author was to be expected. The use of

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[19] Armando Sapori, "Alberti, Antonio," *Dizionario biografico degli Italiani (DBI)* vol. 1 (1960).

Euclid is less so, though of course Euclid's geometry treatise was in more frequent use than the writings of Boethius. Its presence here suggests that features of Boethius's works were seen to have applications broader than their traditional and limited contexts. It also reflected Varchi's own interests. Varchi was working at the time on a vernacular translation of the first books of Euclid, a project he mentioned in a letter to Strozzi in April 1540, although he never published it.[20] His interest in bringing university topics into vernacular language would continue throughout his life; within the year he would become a leading member of a new academy in Padua, the Infiammati, devoted to discussions and lectures on philosophical topics in vernacular.

Varchi begins his treatise with definitions and categories, distinguishing commensurable and incommensurable quantities; square, cubic, prime numbers; and rationals versus irrationals. Then he moves on to the various species of proportion as described by Boethius: multiplex (x:1), superparticular (x+1:x), superpartiente (x+y:x). Varchi stops at multiple points throughout the work to offer sets of rules for generating an infinite series of numbers in a given proportional relation. On occasion (for example, fols. 11v, 18v) he reminds the reader that the presentation is organized with the game in mind, by noting that the given topic under discussion is particularly useful to the game of Pythagoras.

Yet the game, he notes, is hardly the only reason for undertaking a study of proportionality. Varchi reminds his readers that the topic is relevant to speculation in music, astrology, and geometry, valuable for understanding ancient authors, and especially important "for natural calculations treated by Aristotle in the Physics."[21] He repeats the point when he discusses the set of four numbers that players seek to form with their pieces to claim the highest victory (as in 6, 8, 9, 12), the "greatest and most perfect mean." Boethius had said there was nothing more perfect, he notes, and this set of proportions is used both in music and in solving problems in natural philosophy:

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[20] Samuels, "Benedetto Varchi," 221.
[21] "et più alle calculationi naturali trattate da Aristotele nella Fisica.", fol. 19r.

It is extremely useful not only in music but also in natural investigations, and it is called the greatest and most perfect because (as Boethius says), nothing can be found more perfect than this proportionality.[22]

Indeed, he continues, one can derive all the musical proportions from them, including the octave, fifth, tone, and more. (fol. 22v)

Varchi offers an example of interpreting ancient authors from Boethius himself, by noting his analogies between the kinds of means and the types of government. Boethius had likened the arithmetic mean to the republic governed by the few, since the lower numbers had a greater proportion. The harmonic mean is like a government by the elite, because it has the greater proportion in the greater numbers. The geometric mean falls in between, the only genuine proportionality, which is like a government by the *popolo* (fols. 23v-24r). He continues with brief discussions of properties of each mean, noting with the geometric mean the common application (seen in Euclid 9.19) commonly known as the "rule of three" for finding an unknown fourth term.

Strozzi then picks up the topic by introducing the game manual itself. It shares many similarities with the manual Lefèvre had first published some forty years earlier, including dialogue form, but is hardly a mere copy; the most obvious difference is length. Lefèvre's dialogue is brief. It occupies only some four pages at the end of the volume, and one of those is filled mainly with an illustration of the board and pieces. Strozzi expands his presentation to more than 15 folios, with nearly three more devoted to his introduction.

Strozzi's dialogue takes the form of a teacher with two students, similar to Lefèvre's format; Strozzi gives the role of teacher to Cosimo Rucellai. He casts himself as one of the students; the other role is played by a friend, Jacopo di Piero Vettori. Jacopo's father Piero

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[22] "è utilissima non solamente alla Musica ma ancora, nelle quistioni naturali, et si chiama grandissima et perfettissima perche (come dice Boetio) non si può trovare cosa piu perfetta di questa proportionalità." Fol. 21v.

was already well known as a humanist scholar; at this time, he was moving into the career as classics professor that he would hold for decades. Varchi corresponded with both Jacopo and Piero, though more frequently with Piero as he and Varchi worked together on editions of classical texts.

Two other features of Strozzi's manual show clearly his reliance on Lefèvre. One is the dimensions of the game board itself. Most descriptions of rithmomachia call for a doubled chess board, that is, two boards placed end to end for a total of 8 x 16 squares. Strozzi's version follows Lefèvre instead, with a smaller field of 8 x 10 squares. For Lefèvre this may have represented a genuine variation; it is also possible that it simply saved space on the page, given the edition's brevity and its cramped page format.

The other feature is a reference to symbolic values associated with the playing pieces. Carlo asks whether there is any significance to the choice of colors just given. The odd side is colored white, explains Cosimo, because odd numbers are masculine and more perfect, and therefore have the more perfect color; the pyramid on the even side, conversely, is more perfect and gets the more perfect color (red; the other is blue). Lefèvre had developed these themes at somewhat greater length. The teacher, Alcmeon, tells his students that the army of the Evens is black, pertaining to the sensible world, while the army of the odds is white, because the odds are masculine, as if always a single substance. He continues with analogies about the pyramids until his students call him back, saying that they want to know about the game and not symbols.[23] In Strozzi's dialogue, Cosimo simply says that in his opinion this is not very important and so he will not say more about it (fols. 31v-32r).

Strozzi also departs from Lefèvre's model in several ways. He devotes considerable detail to the laying out of playing pieces, the kinds of proportions they represent, and the particulars of those relationships; most of these topics would have been covered in other

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[23] Jordanus, Lefèvre d'Étaples, and Boethius, *In hoc opere contenta. Arithmetica decem libris demonstrata ….. Rithmimachie ludus* (1496), unpag.

sections of Lefèvre's volume. He names the ranks of playing pieces, distinguishing between the first rows as foot soldiers (*compagni o fanti*), the others as leaders or guides (*capi o guide*). He also interrupts the dialogue to present, in the form of a list, a set of rules for play. These features have no exact equivalent in Lefèvre's little treatise and suggest the use of another textual source. Indeed, at several points (fols. 40r, 41v) the details given in the list of rules for play refer to variations made by "Castrense." These general occur in situations where one-piece captures another, and according to Castrense the victorious piece retains its place rather than moving to the square of the captured piece. Once the dialogue resumes, Carlo asks Cosimo about the references to Castrense, and also observes that Cosimo mentions Nicole Oresme. Lefèvre had not known of these moves, replies Cosimo, so they were added later at Varchi's request (fol. 42r).

The most likely candidate for this reference is Paolo di Castro (Paulus Castrensis), a noted late 14th-early 15th-century jurist and professor of law who had worked in a number of places including Florence.[24] It might also refer to his son Angelo.[25] Cosimo notes another variation on the movements of the pyramids that he attributes to Oresme, but, he says, the exact meaning is not clear and so he gives it in Latin. In any case, he concludes, these are the rules that came with the game set originally. They discuss the various sets of proportions and means that each side can produce in victory, which as with most manuals are given in table form. The dialogue ends (46r) with the arrival of the senior Rucellai (46r).

The catchword "Richimachia" at the bottom of that page, below the "il fine" that marks the end of the dialogue, points to yet another set of rules for the game that begin on the next page. These are in Latin and bear neither attribution nor title; they may be copies of the pages that had accompanied Lefèvre's volume and the game set from Paris, as alluded

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[24] On Paolo di Castro, see Susanne Lepsius, "Paolo d Castro as Consultant: Applying and Interpreting Florence's Statues," in The Politics of Law in Late Medieval and Renaissance Italy, ed. Lawrin Armstrong and Julius Kirshner, 77-105 (Toronto: University of Toronto Press. 2011).
[25] On Angelo di Castro see Giuliana D'Amelio, "Castro, Angelo da (Angelo di Castro, Angelo Castrense)," *DBI* Vol. 22 (1979).

to in the introductions. There are some nineteen rules and another eight "cautiones;" the rules are similar to Lefèvre's twelve rules but definitely not identical. They include the references to Castrensis and Oresme, and are presumably their source. No other known references or texts connect either figure with the game. The rules end in turn without ceremony on 49v, and the copyist returns to vernacular to offer a set of rules for finding a missing number in a series, of the sort Varchi discusses. He concludes on 51r with a bilingual "Finis il Fine."

This Florence-centered group of players seems to have extended itself to Rome. Varchi, in his introductory letter asked a favor of Luca Martini relating to Luigi Alamanni, the father of the recipient of Strozzi's introductory letter. Luigi Alamanni was living in exile due to his involvement with the republican government of 1527; he had recently moved to Rome, serving as secretary to cardinal Ippolito d'Este of Ferrara.[26] Upon his arrival, he had worked to convince Paul III to release the artist Benvenuto Cellini from imprisonment in Castel Sant'Angelo. This effort was met with success in December, and Varchi asked Luca Martini in his introductory letter to send Alamanni a gift in gratitude: a copy of the manuscript with a game set made by Giovanni Battista Tasso and painted by Bronzino (fol. 2v).[27] Despite this evidence that the game seemed a worthy gift for such an occasion, nothing further is known about its subsequent popularity in Florence, Rome, or Padua.

For Varchi and his circle of learned friends and students, rithmomachia—or, as they referred to it, Pythagoras—was an engaging and engrossing pastime with a notable pedigree, a worthy form of recreation. Yet none of them maintained a strong or noticeable interest in mathematics throughout their careers, an interest that might correlate with a long-term engagement with the game, and it is not clear that they continued to play it during their later years. Nor did mathematics have much of a profile in Florence's subsequent cultural circles, with their emphasis on letters and vernacular language. It is not

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[26] Samuels, "Benedetto Varchi," 215.
[27] Samuels, "Benedetto Varchi," 216-21.

surprising that some Florentines would have found the game interesting, at least for a time, but equally unsurprising that it never truly became a widespread practice.

Lefèvre's Collège du Cardinal Lemoine had developed a distinctly different community. Founded in 1302, it was intended to house and assist students and scholars from Picardy, as was Lefèvre. Thanks to the research of Richard Oosthoff, the features of this residential scholarly world have become clearer.[28] Under Lefèvre and his colleagues, among them Josse Clichtove and Charles de Bovelles, as well as students such as Beatus Rhenanus, Lemoine featured learning, friendship, religious devotion, and moral development in a residential setting that even produced its own textbooks and study aids. It also emphasized mathematical study in reaching all of these goals. In particular, Lefèvre's Lemoine favored quadrivial learning as developed by Boethius, enriched by humanist texts and learning; as Oosterhoff notes, "… this circle raised the status of Boethian mathematics and developed its logic within practices of university teaching, collaborative authorship, and making books."[29]

The volume in which Lefèvre published his rithmomachia treatise was his earliest publication to promote these efforts. It included an edition of Jordanus Nemorarius's thirteenth-century work on number theory, with Lefèvre's commentary; his *Elementa musicalia*, based on Boethius; and also his Epitome of Boethius's Arithmetic. In multiple works over many years he recommended, along with Josse Clichtove and Charles Bovelles, practical aids to mathematical learning, rithmomachia among them.[30] In his game manual he also emphasized the Pythagorean approaches to mathematics with his choice of interlocutors. The role of magister is named Alcmeon, "mathematician and disciple of Pythagoras" (mathematicus Pythagore discipulus), and his students are Brontinus and

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[28] Richard Oosterhoff, *Making Mathematical Culture: University and Print In the Circle of Lefèvre D'Étaples* (Oxford: Oxford University Press, 2018).

[29] Oosterhoff, *Making Mathematical Culture*, 8.

[30] Oosterhoff, *Making Mathematical Culture*, 88, 96, 132.

Bathillus, "youths of his day" (*eius temporis adolescentes*). Calcidius, in his commentary on Plato's Timaeus, refers to Alcmaeon of Croton as a student of Pythagoras; Alcmaeon dedicated works in turn to fellow Pythagoreans Brontinus of Metapontum and Bathyllus, as mentioned by Diogenes Laertius (8.83).

Even in Paris, this long moment of mathematics education did not last forever. Oosterhoff summarizes the transitions in the following generation, from mathematics as aid to contemplation to mathematics as aid to problem solving.[31] Nonetheless, rithmomachia continued to find a sufficient audience of Parisian players that at midcentury, mathematics teacher Claude de Boissière published his own game manuals in both Latin and French in 1554 and 1556.[32]

Boissière's manual found its own range of readers in turn. The Padua mathematics professor Francesco Barozzi states that he encountered a copy of the Latin edition in Bologna, taught the game to Camillo Paleotti and Francesco Caporacia, and decided to publish his own version in Italian.[33] He also referred to Lefèvre's publication. Although Barozzi lectured in mathematics at Padua, he gave no indication that he had found any interest in the game there. Nor does he reveal any awareness of Varchi and Strozzi's work. Thus, the pursuits of Varchi and his colleagues some decades earlier seem not to have left any network of players there. Barozzi, despite his own interests in Platonic and Pythagorean thought, and his claim in the Proemio that the game had originated in those ancient communities (though, he acknowledged, no Greek exemplar had been discovered), omitted any discussions of Boethian proportion altogether from his work. He did so, he

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[31] Oosterhof, *Making Mathematical Culture*, 214-30.

[32] Claude de Boissière, *Nobilissimus et antiquissimus ludus Pythagoreus, qui Rythmomachia nominatur ....* (Paris: Cavellat, 1554; "nouvellement amplifié," 1556); Boissière, *Le très excellent et ancien Jeu Pythagorique, dit Rithmomachie... pour obtenir vraye et prompte habitude en tout nombre et proportion …* (Paris: s.n., 1556).

[33] Francesco Barozzi, *Il nobilissimo et antiqvissimo givoco Pythagoreo nominato rythmomachia cioe battablia de consonantie de nvmeri: ritrouato per vtilità & solazzo delli stidiosi, et al presente in lingua volgare in modo di paraphrasi composto (*Venice: Perchacino, 1572).
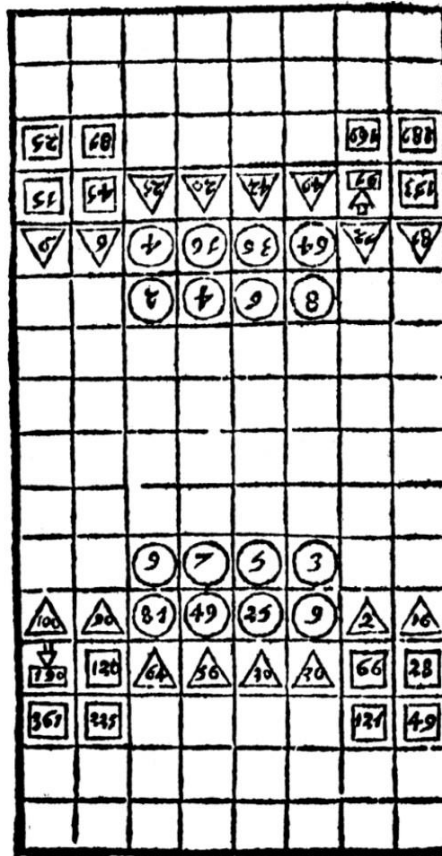
observed, so as not to confuse those who were unfamiliar with the study of proportion and simply wanted to play.[34]

Figure 1

Rafe Lever and William Fulke, *The most noble, auncient, and learned playe, called the Philosophers game*. London: James Rowbothum, 1563, sig. B ii verso.

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[34] Barozzi, *Nobilissimo et antiqvissimo givoco*, 8.

The interest in Boethius's textbooks followed a similar path. The humanist, musical scholar, and professor of theology Heinrich Glarean produced a new edition of the Arithmetic and Music for inclusion in the Basel Opera omnia of 1546, based on a more correct eleventh-century manuscript. The edition was reprinted in 1570, two years before the Barozzi's game manual; thereafter, no new edition of Boethius's quadrivial works appeared until the nineteenth century.[35]

The manual of Varchi and Strozzi thus represents a significant example of the last appearances of rithmomachia and the scholarly world of which it was a part. Yet right up until the end, it found enthusiastic players and a distinguished audience, including some of the leading men of letters and artists of the era.

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[35] Anitii Manlii Seuerini Boethi … Opera, quae extant, omnia … Porrò, infinitis locis non aliquot uoces, sed longas periodas quae in prioribus aeditionibus desiderabantur, emendauimus, adiuti praesidio uetustissimorum manuscriptorum exemplarium, & auxilio doctissimorum uirorum: inter quos in omni literarum genere summus uir Henrichus Loritus Glareanus arithmeticam & musicam demonstrationibus & figuris auctiorem redditam suo pristino nitori restituit, cui exemplar aureum uenerandaeque uetustatis fuit ex monasterio S. Georgij … (Basel: Henricus Petrus, 1546; reprinted, 1570).

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